

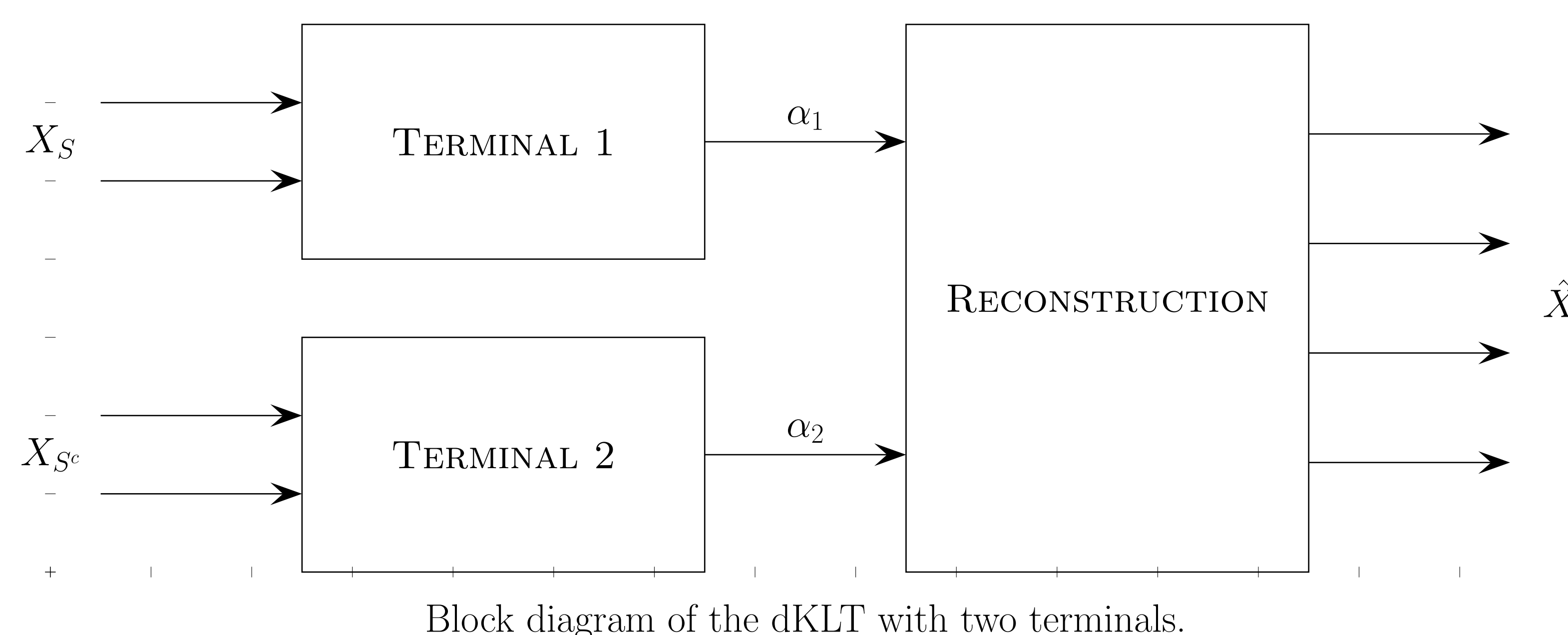
# Asymptotic Distortion Behavior of the Distributed Karhunen-Loève Transform

## Motivation

- Signal approximation in centralized and distributed scenarios.
- Precise understanding of the reconstruction error is crucial:
  - Efficient distributed communication.
  - Robustness to sensor failure.
  - Scaling behavior.
- The distributed Karhunen-Loève Transform (dKLT) [1] plays a fundamental role in this context.
- Goals:
  - Study the asymptotic distortion behavior of the dKLT.
  - Apply our results to a simple correlation model (first-order Gauss-Markov).

## Problem Setup

- Distributed approximation of a jointly Gaussian random source  $X^T = (X_S^T, X_{S^c}^T)$ .
  - Terminal 1: a fraction  $\alpha_1$  of transformed coefficients to describe  $X_S \in \mathbb{R}^n$ .
  - Terminal 2: a fraction  $\alpha_2$  of transformed coefficients to describe  $X_{S^c} \in \mathbb{R}^n$ .
- We want to minimize the mean squared reconstruction error  $E[\|X - \hat{X}\|^2]$ .



- The general problem is difficult:
  - Globally optimal transform unknown to date.
  - Locally optimal solution: dKLT algorithm [1].
  - Distortion not tractable analytically.
- Analysis of the mean squared reconstruction error using the theory of large Toeplitz matrices [2]: asymptotic normalized distortion.
- We study the following cases:
  - Joint processing (terminal 1 and 2 are merged): usual KLT with  $\alpha = (\alpha_1 + \alpha_2)/2$ .
  - Distributed processing:
    - \*  $\alpha_1 = 1$  or  $\alpha_2 = 1$ : conditional KLT (cKLT).
    - \*  $\alpha_1 = 0$  or  $\alpha_2 = 0$ : partial KLT (pKLT).

## Asymptotic Normalized Distortion

- Joint processing: the asymptotic normalized distortion for a stationary process with power spectrum density  $f(\lambda)$  is derived as

$$D(\alpha) = \frac{1}{2\pi} \int_{\lambda: f(\lambda) \leq x_\alpha} f(\lambda) d\lambda$$

where  $F(x)$  denotes the corresponding limiting eigenvalue distribution and  $x_\alpha$  satisfies  $F(x_\alpha) = 1 - \alpha$ .

- Distributed processing: extension to  $D(\alpha_1, \alpha_2)$ . We provide the corresponding formulas for the cKLT ( $D(\alpha_1, 1)$ ) and the pKLT ( $D(\alpha_1, 0)$ ).
- Exact characterization of the gain/loss due to side information/hidden part can thus be provided.

## Example: first-order Gauss-Markov

- The process is obtained as follows:

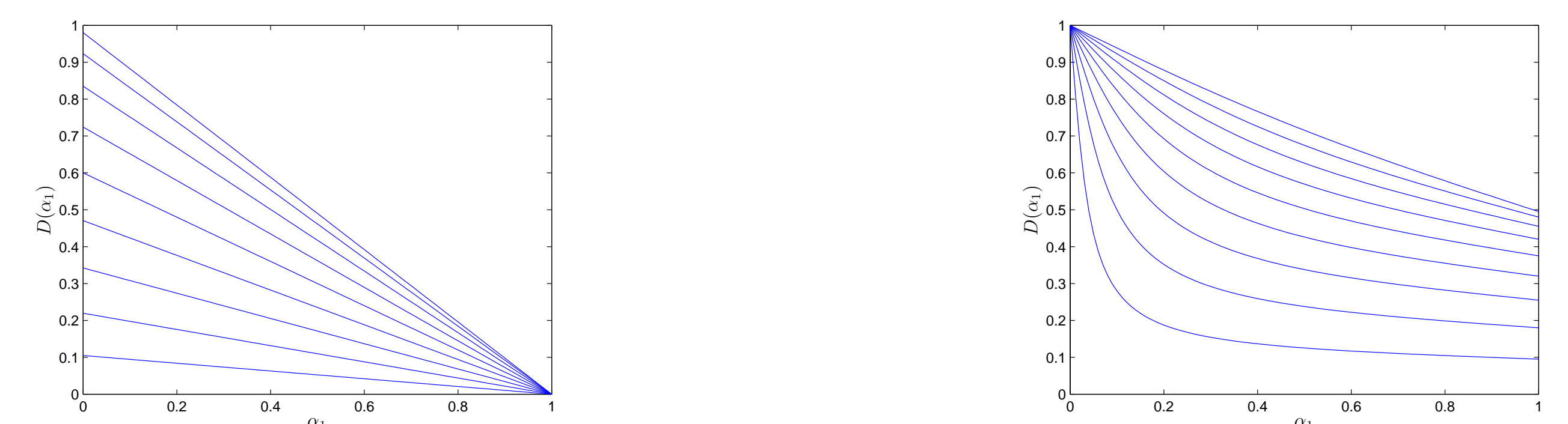
$$X_n = \rho X_{n-1} + Z_n$$

with correlation parameter  $\rho \in (0, 1)$  and  $Z_n$  i.i.d. zero mean Gaussian random variable with variance  $1 - \rho^2$ .

- We consider the following separation: odd samples ( $X_S$ ) and even samples ( $X_{S^c}$ ).
- Joint processing:
  - Closed-form formulas for the asymptotic normalized distortion (left plot) for  $\rho = 0.1, 0.2, \dots, 0.9$  (top to bottom).
  - Finite dimensional approximation (right plot) for  $n = 10$  and  $\rho = 0.5$ .



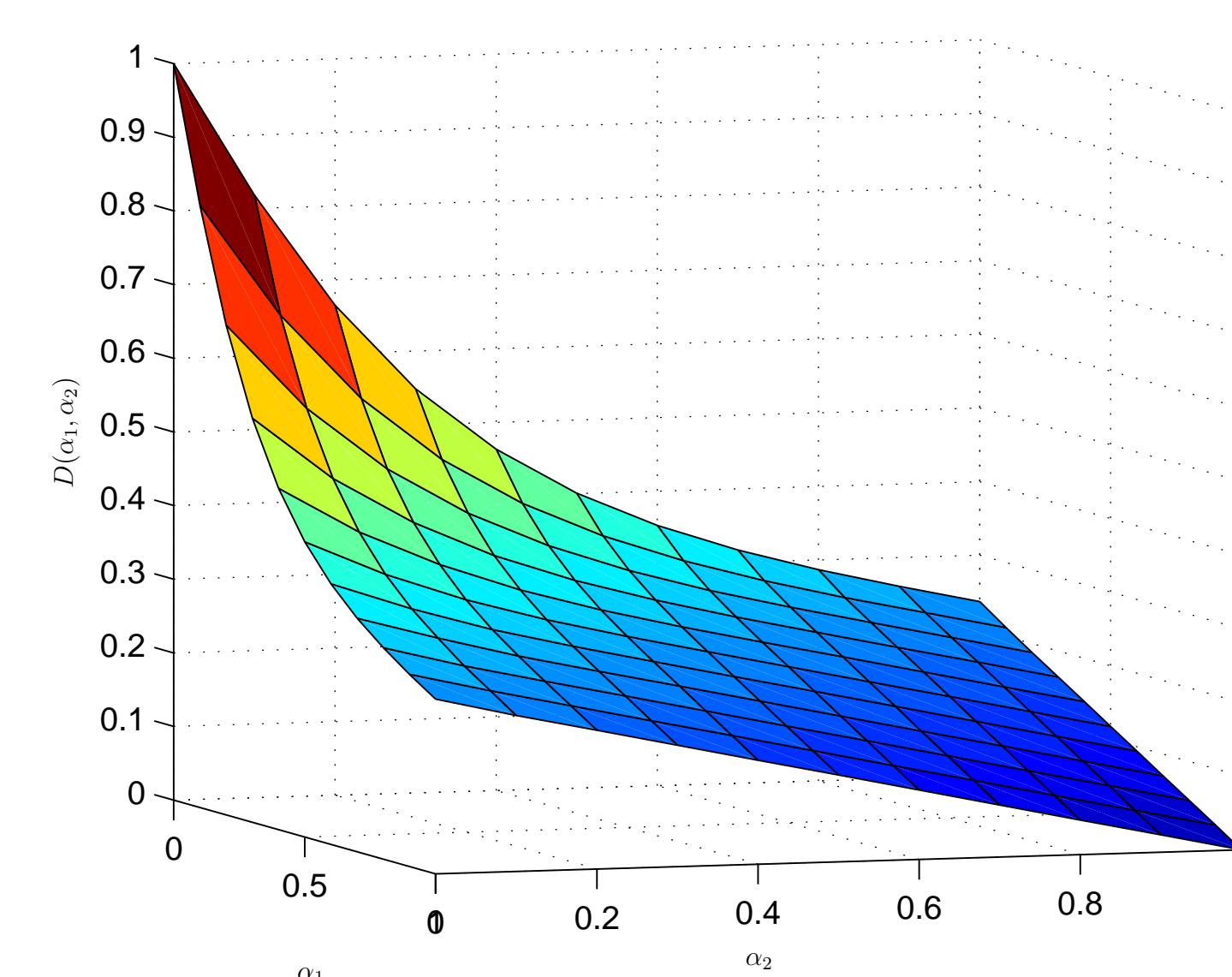
- Distributed processing:
  - Closed-form formulas for the asymptotic normalized distortion of the cKLT (left plot) and the pKLT (right plot) for  $\rho = 0.1, 0.2, \dots, 0.9$  (top to bottom).



- We precisely quantify the gain/loss due to side information (left plot) and hidden part (right plot).



- The above results correspond to the borders of the general asymptotic normalized distortion surface for arbitrary  $\alpha_1$  and  $\alpha_2$  shown below for  $\rho = 0.6$  (numerical experiment).



## References

- [1] M. Gastpar, P. L. Dragotti and M. Vetterli, “The distributed Karhunen-Loève transform”, submitted to IEEE Transactions on Information Theory, November 2004.
- [2] R. M. Gray, “On the asymptotic eigenvalue distribution of Toeplitz matrices”, IEEE Transactions on Information Theory, vol. 18, pp. 725-730, November 1972.